HEAT AND MASS TRANSFER IN A BINARY LAMINAR BOUNDARY LAYER WITH FREE CONVECTION ON A VERTICAL POROUS SURFACE

P. M. Brdlik and I. S. Molchadskii UDC 536.244:536.25

An approximate analytical solution of heat and mass transfer in a binary laminar boundary layer with free convection on a vertical surface is presented. The numerical solution is compared with an approximate analytical solution obtained by another method.

An approximate analytical solution is given in [1] for binary laminar free convection obtained by Squire's method [2], according to which the integral equations of momentum and energy are integrated at the same upper limit equal to the thickness of the thermal boundary layer with the introduction into the equation of motion of an additional function with the dimension of velocity which is a function of the Prandtl number. A different approach is used here: the equations of momentum and energy are integrated at different upper limits, and the ratio of the thermal and diffusion boundary layers are assumed to be equal to $Le^{-1/3}$. In addition, the distribution of velocities, temperature, and concentration are assigned in a different form than in [1].

The integral equations of the binary boundary layer for laminar free convection are written in the following way [1]:

equation of momentum

$$\rho_\infty \frac{d}{dx}\int_0^\delta u^2 dy = -\tau_w + \int_0^h (\rho-\rho_\infty) g dy, \tag{1}$$

equation of energy

$$c_p\rho_\infty \frac{d}{dx}\int_0^{\delta_T} u(t-t_\infty)\,dy = q_w + \rho_w v_w c_p (t_w - t_\infty) + (c_{p_1} - c_{p_2})\int_0^{\delta_T} [j_1]\frac{\partial t}{\partial y}\,dy, \tag{2}$$

equation of diffusion

$$\rho_\infty \frac{d}{dx}\int_0^{\delta_m} u(m_1 - m_{1\infty})\,dy = j_{1w} + \rho_w v_w (m_{1w} - m_{1\infty}). \tag{3}$$

The equations do not take into account compressibility and viscous dissipation, and the properties of the mixture are assumed constant with the exception of a change of density due to the temperature and density in terms with lift.

The porous wall is considered semipermeable: permeable for the active component (component 1) and impermeable for the second component.

Then the transverse velocity v_w is written

$$v_w = \frac{j_{1w}}{\rho_w(1-m_w)}. \tag{4}$$

Scientific-Research Institute of Structural Physics, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 1, pp. 19-28, July, 1971. Original article submitted October 20, 1970.

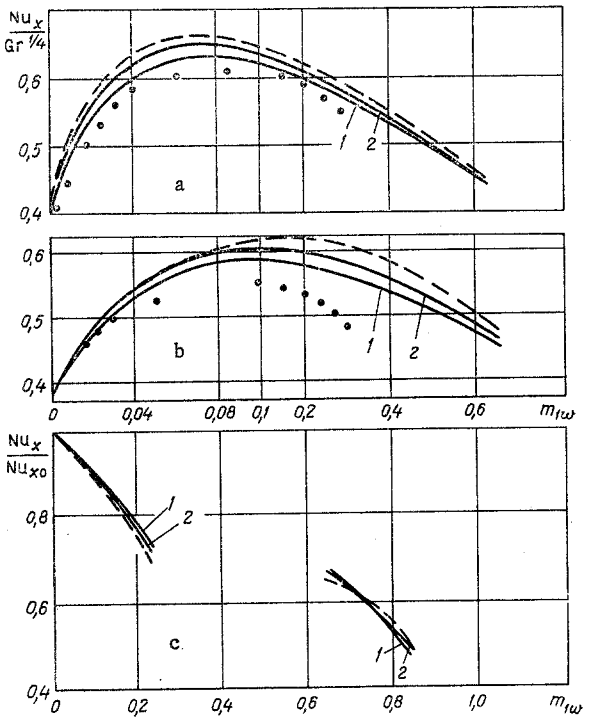

Fig. 1. Heat transfer in the case of injection of hydrogen (a), helium (b), and carbon dioxide (c) into laminar boundary layer of air ($T_w/T_\infty = 1 \cdot 1$; $T_w = 366.4$; dashed curve: numerical calculation [3] (variable physical properties); dots: numerical calculation [3] (constant physical parameters of mixture); 1) approximate solutions [1]; 2) approximate solution of present article): for a) H_2–air, $Sc_w/Pr_\infty = 0.35$–2.5; b) He–air, $Sc_w/Pr_\infty = 0.35$–2.5; c) CO_2–air, $Sc_\infty/Pr_\infty = 1.2$–$1.89$.

The local specific heat flux

$$q = -\lambda\left(\frac{\partial T}{\partial y}\right) + \left(\frac{a_T R M^2 T}{M_1 M_2}\right) j_1. \quad (5)$$

The concentration flux of component 1 (injected gas)

$$j_1 = -\rho D\left(\frac{\partial m_1}{\partial y}\right). \quad (6)$$

According to a theoretical analysis [1, 4], thermal diffusion in the case of free convection can be neglected in the overwhelming majority of applied problems.

The mass relative concentration of injected gas $m_1 = \rho_1/\rho_{mix}$ is determined by

$$m_1 = \frac{P_1 R}{R_1 P_{mix}} = \frac{R_2 P_1}{R_1 P_{mix} - P_1(1 - R_2)},$$

where

$$R = m_1(R_1 - R_2) + R_2.$$

A change of density over the thickness of the boundary layer occurring due to differences of temperatures and concentrations:

$$\frac{\rho - \rho_\infty}{\rho_\infty} = -\beta_T(t - t_\infty) - \beta_m(m_1 - m_{1\infty}). \quad (7)$$

The dimensionless concentration coefficient of volume expansion has the form

$$\beta_m = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial m_1}\right)_{e.t.} \approx \frac{\frac{M_2}{M_1} - 1}{1 + \left(\frac{M_2}{M_1} - 1\right) m_1}. \quad (8)$$

Equations (1)-(3) with consideration of (6) and (7) have the form

$$\frac{d}{dx}\int_0^{\delta} u^2 dy = g(T_\infty - T_w)\beta_t \int_0^{\delta_t} \theta dy - g\beta_m \int_0^{\delta_m} \bar{m} dy - \nu\left(\frac{\partial u}{\partial y}\right)_w, \quad (9)$$

$$\frac{d}{dx}\int_0^{\delta_t} \theta u dy = -a\left(\frac{\partial\theta}{\partial y}\right)_w - \mathrm{Du}\,\bar{\rho} D\left(\frac{\partial\bar{m}}{\partial y}\right)_w + \frac{D}{1 - m_{1w}}\,\bar{\rho}\left(\frac{\partial\bar{m}}{\partial y}\right)_w + \frac{c_{p_1} - c_{p_2}}{c_p}\,\bar{\rho} D \int_0^{\delta_T} \frac{\partial\bar{m}}{\partial y}\cdot\frac{\partial\theta}{\partial y}\,dy, \quad (10)$$

$$\frac{d}{dx}\int_0^{\delta_m} \bar{m} u dy = -\frac{1 - m_{1\infty}}{1 - m_{1w}}\,\bar{\rho} D\left(\frac{\partial\bar{m}}{\partial y}\right)_w, \quad (11)$$

where

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}; \quad \bar{m} = m_1 - m_{1\infty}.$$

The following distributions are used in calculating the integrals in (9)-(11):

$$u = a_1 + b_1 y + c_1 y^2 + d_1 y^3,$$
$$m = a_2 + b_2 y + c_2 y^2 + d_2 y^3, \quad (12)$$
$$T = a_3 + b_3 y + c_3 y^2 + d_3 y^3.$$

TABLE 1

Relationships of boundary layers: $\delta, \delta_t, \delta_m$	Expressions for determining quantity $\sigma = \delta/\delta_t$	Expressions for determining quantity $\sigma = \delta/\delta_t$
1. $\delta_t > \delta > \delta_m$ $Sc > 1$; $Pr < 1$; $Le < 1$	$\sigma^3 (0{,}0177HA_1 - 0{,}0715A_1) + \sigma^2 [Le^{2/3} \left(0{,}6 - 2l_1 + \frac{9}{20} l_2 \right)$ $+ 0{,}0212A_1 - 0{,}0098HA_1] + \sigma [Le \left(3l_1 - \frac{5}{6} + 0{,}7l_2 \right) - 0{,}00rA_1$ $+ 0{,}0016HA_1] - Le^{4/3} \left(\frac{9}{28} - 1{,}2l_1 - \frac{2}{7} l_2 \right) = 0;$	Values of quantity $H = \frac{1}{\varkappa_1} \left(6 + \frac{AT_w}{T_w - T_\infty} \right);$ $A_1 = \left\{ 6 \frac{1 - m_{1\infty}}{1 - m_{1w}} l_1 Le^{-1/3} \bar{\rho} \right\} \Big/ \left\{ Le^{-1}H + Le^{-1/3} \bar{\rho} (m_{1w} - m_{1\infty}) 6l_1 \right.$
2. $\delta_t > \delta_m > \delta$ $Sc < 1$; $Pr < 1$; $Le < 1$	$\sigma^3 \left[Le^{-1} \left(\frac{1}{70} l_2 + \frac{1}{210} \right) - A_1 \left(\frac{2}{105} - \frac{H}{105} \right) \right] + \sigma^2 \left[\frac{1}{20} l_2 Le^{-2/3} \right.$ $\left. + A_1 \left(\frac{H}{30} - \frac{1}{20} \right) \right] + \sigma \left(\frac{HA_1}{30} - \frac{1}{5} l_1 Le^{-1/3} \right) - \left(\frac{1}{12} - \frac{A_1}{12} \right) = 0;$	$\times \left(Du + \frac{1}{1 - m_{1w}} \right) - \frac{c_{p_1} - c_{p_2}}{c_p} \bar{\rho} \left\langle \frac{6 (T_\infty - T_w)(m_{1\infty} - m_{1w})}{4\bar{\rho} + Pe_g} \right.$ $\times \left\{ H \left[\bar{\rho} (1 - Le^{1/3})^2 + Pe_g \left(\frac{1}{6} - \frac{1}{3} Le^{1/3} + \frac{3}{20} Le^{2/3} \right) \right] + \bar{\rho} \, (3Le^{1/3} \right.$
3. $\delta_m > \delta_t > \delta$ $Sc < 1$; $Pr < 1$; $Le > 1$	$\sigma^3 \left[Le^{-1} \left(\frac{1}{70} l_2 + \frac{1}{210} \right) - B \left(\frac{2}{105} - \frac{H}{105} \right) \right] + \sigma^2 \left[\frac{1}{20} l_2 Le^{-2/3} \right.$ $\left. + B \left(\frac{H}{30} - \frac{1}{20} \right) \right] + \sigma \left[\frac{HB}{30} - \frac{1}{5} l_1 Le^{-1/3} \right] - \left(\frac{1}{12} - \frac{B}{12} \right) = 0;$	$\left. \left. - 2Le^{2/3}) + Pe_g \left(\frac{1}{2} Le^{1/3} - 0{,}3 Le^{2/3} \right) \right\} \right\rangle \Big\};$ $l_1 = \frac{\bar{\rho}}{4\bar{\rho} + Pe_g}; \quad l_2 = \frac{Pe_g}{4\bar{\rho} + Pe_g};$
4. $\delta > \delta_t > \delta_m$ $Sc > 1$; $Pr > 1$; $Le < 1$	$\sigma^2 \left[\left(\frac{3}{20} - \frac{1}{5\varkappa_1} - \frac{1}{30} \cdot \frac{A}{\varkappa_1} \cdot \frac{T_w}{T_w - T_\infty} \right) - A_1^{-1} \left(\frac{3}{5} - 2l_1 \right. \right.$ $\left. \left. - \frac{9}{20} l_2 \right) \right] + \sigma \left[\left(- \frac{2}{15} + \frac{1}{5\varkappa_1} + \frac{1}{30} \cdot \frac{A}{\varkappa_1} \cdot \frac{T_w}{T_w - T_\infty} \right) - A_1^{-1} \right.$ $\left. \times \left(3l_1 - \frac{5}{6} + 0{,}7l_2 \right) \right] + \left[\left(\frac{1}{28} - \frac{2}{35\varkappa_1} - \frac{1}{105} \cdot \frac{A}{\varkappa_1} \cdot \frac{T_w}{T_w - T_\infty} \right) \right.$ $\left. - A_1^{-1} \left(\frac{9}{28} - \frac{6}{5} l_1 - \frac{2}{7} l_2 \right) \right] = 0;$	$A = 6 \frac{a_t RM^2}{M_1 M_2 c_p} \bar{\rho} \, (m_{1w} - m_{1\infty}) \, l_2 Le^{1/3};$ $B = \left[6 \bar{\rho} \frac{1 - m_{1\infty}}{1 - m_{1w}} l_1 Le^{-1/3} \right] \Big/ \left[Le^{-1}H + 6\bar{\rho} Le^{-1/3} (m_{1w} - m_{1\infty}) \, l_1 \right.$ $\times \left(Du + \frac{1}{1 - m_{1w}} \right) - \frac{c_{p_1} - c_{p_2}}{c_p} \bar{\rho} Le^{-1/3}$ $\times \left\{ \frac{6 (T_\infty - T_w)(m_{1\infty} - m_{1w})}{4\bar{\rho} + Pe_g} \left[H \, Pe_g \, Le^{-1/3} \left(\frac{1}{15} Le^{-1/3} - \frac{1}{12} \right) \right. \right.$
5. $\delta > \delta_m > \delta_t$ $Pr > 1$; $Sc > 1$; $Le > 1$	$\sigma^2 \left[\left(\frac{3}{20} - \frac{1}{5\varkappa_1} - \frac{1}{30} \cdot \frac{A}{\varkappa_1} \cdot \frac{T_w}{T_w - T_\infty} \right) - B^{-1} \left(\frac{3}{5} - 2l_1 \right. \right.$ $\left. \left. - \frac{9}{20} l_2 \right) \right] + \sigma \left[\left(- \frac{2}{15} + \frac{1}{5\varkappa_1} + \frac{1}{30} \cdot \frac{A}{\varkappa_1} \cdot \frac{T_w}{T_w - T_\infty} \right) - B^{-1} \right.$ $\left. \times \left(3l_1 - \frac{5}{6} + 0{,}7l_2 \right) \right] + \left[\left(\frac{1}{28} - \frac{2}{35\varkappa_1} - \frac{1}{105} \cdot \frac{A}{\varkappa_1} \cdot \frac{T_w}{T_w - T_\infty} \right) \right.$ $\left. - B^{-1} \left(\frac{9}{28} - \frac{6}{5} l_1 - \frac{2}{7} l_2 \right) \right] = 0;$	$\left. \left. + \bar{\rho} + Pe_g \, Le^{-1/3} \left(\frac{1}{2} - 0{,}3 \, Le^{-1/3} \right) \right] \right\} \Big];$ $\varkappa_1 = 4 + Le^{2/3} Pe_g \left[1 + 6 \frac{c_{p_1} - c_{p_2}}{c_p} \bar{\rho} \, (m_{1w} - m_{1\infty}) \, l_1 \frac{1}{Pe_g} \right];$ $a = 4g\beta_t (T_\infty - T_w) \left(\frac{1}{2} - \frac{1}{12} H \right) Sc + 4g\beta_m (m_{1\infty} - m_{1w}) \, Le^{1/3}$ $\times \left(\frac{9}{8} - 3l_1 - \frac{5}{6} l_2 \right) Sc;$
6. $\delta_m > \delta > \delta_t$ $Sc < 1$; $Pr > 1$; $Le > 1$	$\sigma^4 \left(\frac{1}{70} l_2 + \frac{1}{210} \right) Le^{-1} + \sigma^3 \left[a \left(\frac{1}{70} l_2 + \frac{1}{210} \right) Le^{-1} - \frac{1}{20} \right.$ $\left. \times l_2 Le^{-2/3} \right] - \sigma^2 \left(\frac{6}{63} \Psi \bar{\rho} \frac{1 - m_{1\infty}}{1 - m_{1w}} l_1 Le^{-1/3} + \frac{a}{20} l_2 Le^{-2/3} \right.$ $\left. + \frac{b}{15} l_1 Le^{-1/3} \right) - \sigma \left(\frac{a}{5} l_1 Le^{-1/3} - \frac{b}{12} \right) + \frac{a}{12} = 0;$	$b = g\beta_t (T_\infty - T_w) \left(\frac{1}{2} - \frac{1}{12} H \right) Pe_g \, Le^{-1/3} + g\beta_m (m_{1\infty} - m_{1w})$ $\times \left(\frac{9}{8} - 3l_1 + \frac{5}{8} l_2 \right) Pe_g - \Psi Sc;$ $\Psi = g \, [\beta_t (T_\infty - T_w) + \beta_m (m_{1\infty} - m_{1w})]$

In finding the velocity distribution it was taken into account that when $y = 0$:

$$u = 0, \quad v_w \left(\frac{\partial u}{\partial y}\right)_w = [g\beta_t (T_\infty - T_w) + g\beta_m (m_{1\infty} - m_{1w})] + \nu \left(\frac{\partial^2 u}{\partial y^2}\right)_w,$$

when $y \to \infty$

$$u = 0, \quad \frac{\partial u}{\partial y} = 0.$$

This gives the following velocity distribution over the boundary layer thickness:

$$u = \frac{\psi}{D} \cdot \frac{\delta^2}{4\mathrm{Sc} + \mathrm{Pe}_g \dfrac{\delta}{\delta_m}} \cdot \frac{y}{\delta} \left(1 - \frac{y}{\delta}\right)^2, \tag{13}$$

where

$$\mathrm{Pe}_g = \frac{v_w \delta_m}{D};$$

for injection ($m_{1W} > m_{1\infty}$) and for $T_W > T_\infty$:

$$\psi = g\left[\beta_t (T_w - T_\infty) + \beta_m (m_{1w} - m_{1\infty})\right];$$

for suction ($m_{1W} < m_{1\infty}$) and for $T_W < T_\infty$:

$$\psi = g\left[\beta_t (T_\infty - T_w) + \beta_m (m_{1\infty} - m_{1w})\right].$$

The concentration distribution has the form

$$m_1 - m_{1\infty} = (m_{1w} - m_{1\infty}) \left[1 - \frac{6\bar\rho}{4\bar\rho + \mathrm{Pe}_g} \cdot \frac{y}{\delta_m} - 3\frac{\mathrm{Pe}_g}{4\bar\rho + \mathrm{Pe}_g}\left(\frac{y}{\delta_m}\right)^2 + \frac{1}{2}\left(\frac{y}{\delta_m}\right)^3 + \frac{3}{2} \cdot \frac{\mathrm{Pe}_g}{4\bar\rho + \mathrm{Pe}_g}\left(\frac{y}{\delta_m}\right)^3\right] \tag{14}$$

provided that when $y = 0$

$$m_1 = m_{1w}, \quad \frac{j_{1w}}{\rho_w (1 - m_{1w})} \left(\frac{\partial m_1}{\partial y}\right)_w = \bar\rho D \left(\frac{\partial^2 m_1}{\partial y^2}\right)_w; \tag{15}$$

when $y \to \infty$

$$m_1 = m_{1\infty}, \quad \frac{\partial m_1}{\partial y} = 0.$$

Using Eqs. (4) and (6), we can write the transverse velocity v_W

$$v_w = -\frac{D\left(\dfrac{\partial m_1}{\partial y}\right)_w}{1 - m_{1w}}, \tag{16}$$

and from (14)

$$\left(\frac{\partial m_1}{\partial y}\right)_w = (m_{1w} - m_{1\infty}) \left(-\frac{6\bar\rho}{4\bar\rho + \mathrm{Pe}_g} \cdot \frac{1}{\delta_m}\right). \tag{17}$$

With consideration of (16), (17)

$$\mathrm{Pe}_g = \frac{v_w \delta_m}{D} = \frac{(m_{1w} - m_{1\infty})}{(1 - m_{1w})} \cdot \frac{6\bar\rho}{(4\bar\rho + \mathrm{Pe}_g)},$$

from where after easy transformations we can determine

$$\mathrm{Pe}_g = -2\bar\rho \left[1 - \sqrt{1 - \frac{3}{2} \cdot \frac{1}{\bar\rho} \cdot \frac{m_{1\infty} - m_{1w}}{1 - m_{1w}}}\right], \tag{18}$$

where $\bar{\rho} = \rho_w/\rho_\infty$. We see from (18) that Pe_g does not depend on x if m_{1w} does not depend on x. The following boundary conditions are used for obtaining the temperature distribution over the boundary layer thickness:

when y = 0

$$T = T_w,\quad v_w \left(\frac{\partial T}{\partial y}\right)_w + \frac{c_{p_1} - c_{p_2}}{c_p}\bar{\rho}\left[-D\left(\frac{\partial m_1}{\partial y}\right)_w\right]\left(\frac{\partial T}{\partial y}\right)_w$$
$$= a\left(\frac{\partial^2 T}{\partial y^2}\right)_w + \frac{a_T R M^2 T_w}{M_1 M_2 c_p}\bar{\rho} D\left(\frac{\partial^2 m_1}{\partial y^2}\right)_w, \tag{19}$$

when $y \to \infty$

$$T = T_\infty,\quad \frac{\partial T}{\partial y} = 0.$$

Thus the temperature distribution has the form

$$\frac{T - T_\infty}{T_w - T_\infty} = \left(1 - 3\frac{y^2}{\delta_t^2} + 2\frac{y^3}{\delta_t^3}\right) - \frac{6}{\varkappa}\cdot\frac{y}{\delta_t}\left(1 - \frac{y}{\delta_t}\right)$$
$$- \frac{A}{\varkappa}\cdot\frac{T_w}{T_w - T_\infty}\cdot\frac{y}{\delta_T}\left(1 - \frac{y}{\delta_T}\right)^2, \tag{20}$$

where

$$\varkappa = 4 + \mathrm{Le}\,\frac{\delta_t}{\delta_m}\,\mathrm{Pe}_g\left[1 + \frac{c_{p_1} - c_{p_2}}{c_p}\bar{\rho}\,(m_{1w} - m_{1\infty})\,\frac{6\bar{\rho}}{(4\bar{\rho} + \mathrm{Pe}_g)\,\mathrm{Pe}_g}\right], \tag{21}$$

$$A = \frac{a_T R M^2 \bar{\rho}}{M_1 M_2 c_p}(m_{1w} - m_{1\infty})\frac{6\mathrm{Pe}_g}{(4\bar{\rho} + \mathrm{Pe}_g)}\,\mathrm{Le}\left(\frac{\delta_t}{\delta_m}\right)^2. \tag{22}$$

Integration of Eqs. (9)-(11) is done with consideration of distributions (13), (14), (20), which gives $\delta = c_u x^{1/4}$, $\delta_t = c_t x^{1/4}$, $\delta_m = c_m x^{1/4}$, where the parametric constants c_u, c_t, c_m, are found from the solutions of systems of algebraic equations. The algebraic equations are from the seventh to the fifteenth order, which does not permit determining the parametric constants c_u, c_t, c_m analytically. For an analytical determination of c_u, c_t, and c_m we will assume that the ratio $\delta_t/\delta_m = \mathrm{Le}^{-1/3}$. Substituting the values of j_{iw} into the expression of convective (without consideration of thermal phase transformations of chemical reactions) heat flux on the wall, we obtain

$$q_w = -\lambda\left(\frac{\partial T}{\partial y}\right)_w - \frac{a_t R M^2 T_w}{M_1 M_2}\rho_w D\left(\frac{\partial m_1}{\partial y}\right)_w. \tag{23}$$

The values of $(\partial T/\partial y)_w$ and $(\partial m_1/\partial y)_w$ are determined from distributions (14), (20):

$$\left(\frac{\partial T}{\partial y}\right)_w = \frac{6(T_\infty - T_w)}{\delta_t \varkappa} - \frac{A T_w}{\delta_t \varkappa}, \tag{24}$$

$$\left(\frac{\partial m_1}{\partial y}\right)_w = 6 l_1 \frac{m_{1\infty} - m_{1w}}{\delta_m}, \tag{25}$$

where

$$l_1 = \frac{\bar{\rho}}{4\bar{\rho} + \mathrm{Pe}_g}. \tag{26}$$

Substituting (24) and (25) into (23) and replacing δ_m by $(\delta_t \mathrm{Le}^{1/3})$, after easy transformations we obtain

$$\frac{\alpha}{\lambda} = \frac{6}{\delta_t \varkappa}\left[1 + \frac{A T_w}{6(T_w - T_\infty)} + \mathrm{Du}\,\bar{\rho} l_1 (m_{1w} - m_{1\infty})\,\mathrm{Le}^{2/3}\varkappa\right]. \tag{27}$$

The local mass-transfer coefficient α_m (with consideration of Stefan flux) is found from the expression for the total mass flux of component 1

$$W_{1w} = j_{1w} + \rho_w v_w m_{1w} = \rho_w \alpha_m (m_{1w} - m_{1\infty}). \tag{28}$$

Using Eqs. (6), (16), and (25) and substituting them into (28), we obtain

$$\frac{\alpha_m}{D} = \frac{6l_1}{\delta_m(1-m_{1w})}. \tag{29}$$

From the solution of the integral equations of motion of a binary laminar boundary layer (9), which is general for any relationship of boundary layers, we find the expression for the hydrodynamic, thermal, and diffusion boundary layers:

$$\frac{\delta}{x} = 3.2\ \mathrm{Gr}_{kx}^{-1/4}(4+\mathrm{Re}_{wx})^{1/4}\left\{\frac{1}{\Psi}\left[g\beta_t(T_\infty - T_w)\frac{1}{\sigma}\left(\frac{1}{2}-\frac{1}{12}H\right)\right.\right.$$
$$\left.\left.+ g\beta_m(m_{1\infty}-m_{1w})\frac{1}{\eta}\left(\frac{9}{8}-3l_1-\frac{5}{8}l_2\right)\right](4+\mathrm{Re}_{wx})-1\right\}^{1/4}, \tag{30}$$

$$\frac{\delta_\tau}{x} = 3{,}2\mathrm{Gr}_{kx}^{-1/4}(4+\mathrm{Re}_{wx})^{1/4}\frac{1}{\sigma}\left\{\frac{1}{\Psi}\left[g\beta_t(T_\infty - T_w)\frac{1}{\sigma}\left(\frac{1}{2}-\frac{1}{12}H\right)\right.\right. \tag{31}$$
$$\left.\left.+ g\beta_m(m_{1\infty}-m_{1w})\frac{1}{\eta}\left(\frac{9}{8}-3l_1-\frac{5}{8}l_2\right)\right](4+\mathrm{Re}_{wx})-1\right\}^{1/4},$$

$$\frac{\delta_m}{x} = 3.2\mathrm{Gr}_{kx}^{-1/4}(4+\mathrm{Re}_{wx})^{1/4}\frac{1}{\eta}\left\{\frac{1}{\Psi}\left[g\beta_t(T_\infty - T_w)\frac{1}{\sigma}\left(\frac{1}{2}-\frac{1}{12}H\right)\right.\right.$$
$$\left.\left.+ g\beta_m(m_{1\infty}-m_{1w})\frac{1}{\eta}\left(\frac{9}{8}-3l_1-\frac{5}{8}l_2\right)\right](5+\mathrm{Re}_{wx})-1\right\}^{1/4}. \tag{32}$$

Substituting (31) into (27) and (32) into (29), we have:

$$\mathrm{Nu}_x = 1.87\mathrm{Gr}_{kx}^{1/4}(4+\mathrm{Re}_{wx})^{-1/4}\frac{\sigma}{\varkappa_1}\left[1+\frac{AT_w}{6(T_w-T_\infty)}+\mathrm{Du}\bar{\rho}\,l_1(m_{1w}\right.$$
$$\left.-m_{1\infty})\,\mathrm{Le}^{2/3}\varkappa_1\right]\left\{\frac{1}{\Psi}\left[g\beta_\tau(T_\infty-T_w)\frac{1}{\sigma}\left(\frac{1}{2}-\frac{1}{12}H\right)\right.\right. \tag{33}$$
$$\left.\left.+ g\beta_m(m_{1\infty}-m_{1w})\frac{1}{\eta}\left(\frac{9}{8}-3l_1-\frac{5}{8}l_2\right)\right](4+\mathrm{Re}_{wx})-1\right\}^{-1/4},$$

$$\mathrm{sh}_x = 1.87\mathrm{Gr}_{kx}^{1/4}(4+\mathrm{Re}_{wx})^{1/4}\frac{l_1\eta}{(1-m_{1w})}\left\{\frac{1}{\Psi}\left[g\beta_t(T_\infty-T_w)\frac{1}{\sigma}\right.\right.$$
$$\left.\left.\times\left(\frac{1}{2}-\frac{1}{12}H\right)+g\beta_m(m_{1\infty}-m_{1w})\frac{1}{\eta}\left(\frac{9}{8}-3l_1-\frac{5}{8}l_2\right)\right](4+\mathrm{Re}_{wx})-1\right\}^{-1/4}. \tag{34}$$

The relation between the Sherwood and Nusselt numbers is determined by the following relationship:

$$\frac{\mathrm{sh}_x}{\mathrm{Nu}_x} = \frac{l_1\mathrm{Le}^{-1/3}\varkappa_1}{(1-m_{1w})\left[1+\frac{AT_w}{6(T_w-T_\infty)}+\mathrm{Du}\bar{\rho}l_1(m_{1w}-m_{1\infty})\mathrm{Le}^{2/3}\varkappa_1\right]}. \tag{35}$$

The expression for determining the quantity $\sigma = \delta/\delta_t$ for different cases of the relationship of the hydrodynamic δ, mass-transfer δ_m, and thermal δ_t boundary layers are given in Table 1. The average values of the Sherwood and Nusselt numbers are obtained by multiplying their local values by 4/3.

In Fig. 1a-c the approximate solution is compared with the numerical solution of Gill et al. [3] and with the approximate analytical solution from [1]. The numerical calculation [3] was made for cases when: a) the variation of the physical parameters of the mixture was taken into account; b) the parameters of the mixture were assumed constant with the exception of a change of density of the mixture as a function of temperature and concentration. The calculations were made for $T_w/T_\infty = 1.1$; $T_\infty = 333°K$. In [1] the Dufour and Soret effects were taken into account; the numerical calculation [3] was made without consideration of these effects. In the case of injection of hydrogen and helium the present approximate solution gives a closer value to the numerical calculation than the solution in [1]; however, the divergence between the two approximate solutions does not exceed 5%. The divergence is explained by the fact that the solution in [1] was obtained under the condition of equality of the thickness of the dynamic and thermal boundary layer. In the given solution the ratio $\delta/\delta_t = \sigma$ is a function of the physical parameters of the components of the mixture and concentration of active component 1 and is found from the appropriate equations. In the case of injection of CO_2 the results practically coincide. The curves for CO_2 in Fig. 1c are interrupted.

This occurs owing to the fact that the lifting forces caused by grad m_1 act toward opposite sides (when $T_w/T_\infty > 1$). For

$$|-0.2| < [\beta_m(m_{1w} - m_{1\infty}) + \beta_t(T_w - T_\infty)] < 0.008$$

system of Eqs. (1)-(3) does not have a solution, since unsteady motion and inversion of the boundary layer occur in the boundary layer. For $[(\beta_m(m_{1w} - m_{1\infty}) + \beta_t(T_w - T_\infty)] < -0.2$ it is necessary to change from the solution of equations with the lower edge to the solution with the upper edge of the plate. Thus, the approximate solutions obtained by Squire's method [1] ($\delta = \delta_t$) and in the present study ($\delta_t/\delta_m = Le^{-1/3}$) give practically coinciding results. In addition, the differences in the distributions of velocity, temperature, and concentration adopted in [1] and in the present article do not have any noticeable effect on the exit characteristics of the boundary layer.

NOTATIONS

ρ is the density;
u is the velocity in direction x;
x is the distance along surface;
y is the distance perpendicular to surface;
τ is the tangential stress;
g is the gravitational acceleration;
c_p is the specific heat of mixture;
T is the temperature;
q is the heat flux;
v is the velocity in direction y;
j is the mass flux;
m is the mass concentration;
α_t is the dimensionless thermal diffusion constant;
R is the gas constant of mixture;
M is the molecular weight of mixture;
D is the diffusion coefficient;
β_t is the temperature coefficient of expansion;
β_m is the concentration coefficient of expansion;
$Pe_g = (v_w \delta_m)/D$ is the Peclet number;
$Du = (\alpha_t \cdot RM^2 Tw)/M_1 M_2 C_p(T_w - T_\infty)$ is the Dufour number;
$Gr_k = (qx^3[(\beta_t \cdot (T_\infty - T_w) + \beta_m(m_{1\infty} - m_{1w})])/\nu^2$ is the combined Grashof number;
$Pr = \nu/a$ is the Prandtl number;
$Re = v_w \delta/\nu$ is the Reynolds number;
$Sc = \nu/D$, is the Schmidt number;
$Sh_x = \alpha_m x/D$ is the Sherwood number;
$Nu_x = \alpha x/\lambda$ is the Nusselt number;
a is the thermal diffusivity;
ν is the kinematic viscosity,
δ is the hydrodynamic boundary layer;
δ_m is the concentration boundary layer;
δ_t is the temperature boundary layer.

Subscripts

1 refers to the injected gas;
w refers to the plate surface;
∞ refers to an infinite distance from the plate;
0 refers to an impermeable surface;
x refers to the local values.

LITERATURE CITED

1. P. M. Brdlik, Inzh.-Fiz. Zh., 16, No. 6 (1969).
2. S. Goldstein, Modern Developments in Fluid Dynamics (1938).
3. W. Gill, E. Del Casal, and D. Zen, Inst. J. Heat and Mass Transfer, 8, No. 8 (1965).
4. Sparrow, Minkovic, and Eckert, Teploperedacha, No. 4 (1964).